Warsaw University of Technology

Faculty of Power and Aeronautical Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

Damping matrix

11.2021

$$[M]_{NDOF \times NDOF} \cdot \{\ddot{q}\}_{NDOF \times 1} + [C]_{NDOF \times NDOF} \cdot \{\dot{q}\}_{NDOF \times 1} + [K]_{NDOF \times NDOF} \cdot \{q(t)\}_{NDOF \times 1} = \{F(t)\}_{NDOF \times 1}$$

- $[M]$ (NDOF × NDOF): MASS MATRIX
- $\{\ddot{q}\}$ (NDOF × 1): VECTOR OF NODAL ACCELERATIONS
- $[C]$ (NDOF × NDOF): DAMPING MATRIX
- $\{\dot{q}\}$ (NDOF × 1): VECTOR OF NODAL VELOCITIES
- $[K]$ (NDOF × NDOF): STIFFNESS MATRIX
- $\{q(t)\}$ (NDOF × 1): VECTOR OF NODAL DISPLACEMENTS
- $\{F(t)\}$ (NDOF × 1): LOAD VECTOR

DAMPING MATRIX

Rayleigh model

$$[C] = \alpha \cdot [M] + \beta \cdot [K]$$

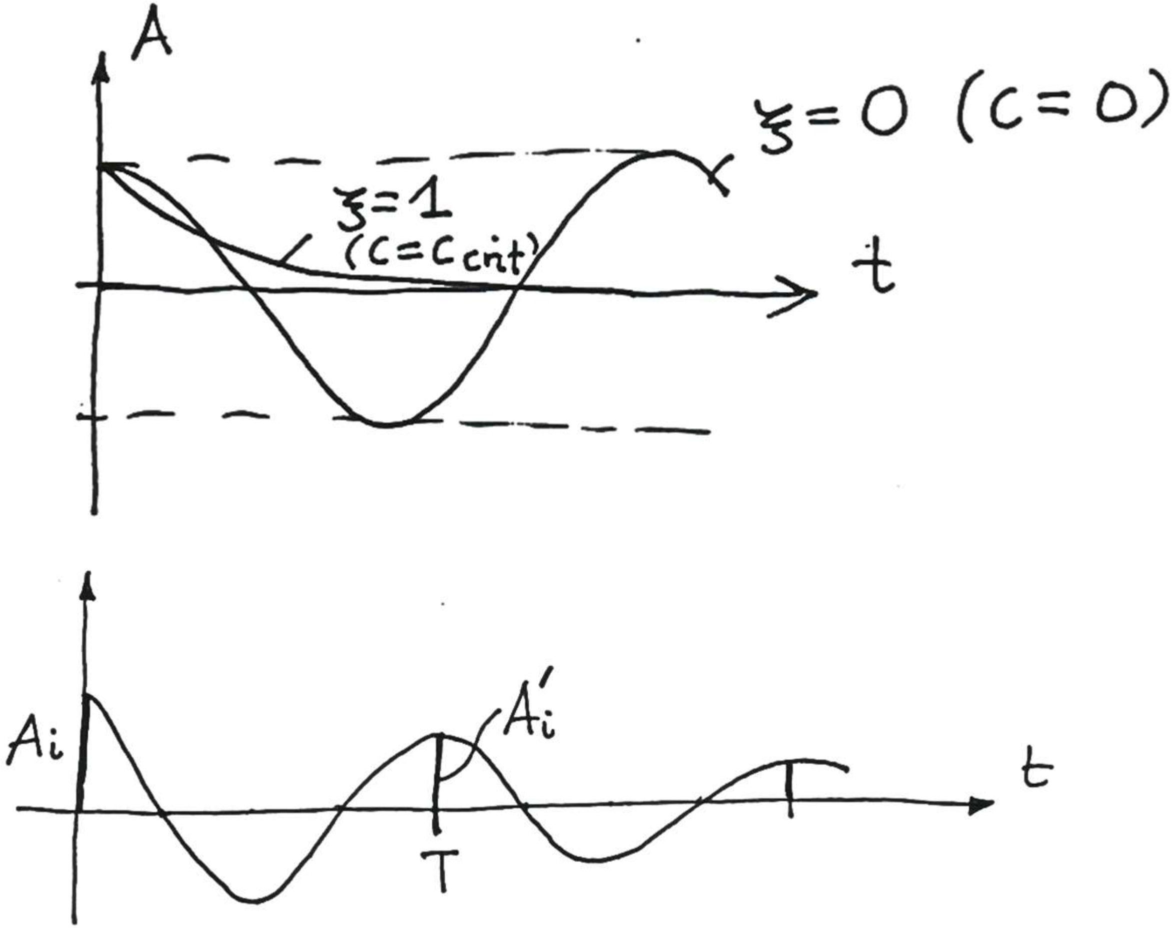
α ↑ coefficient of mass damping (due to rigid body motion)

β ↑ coefficient of structural damping

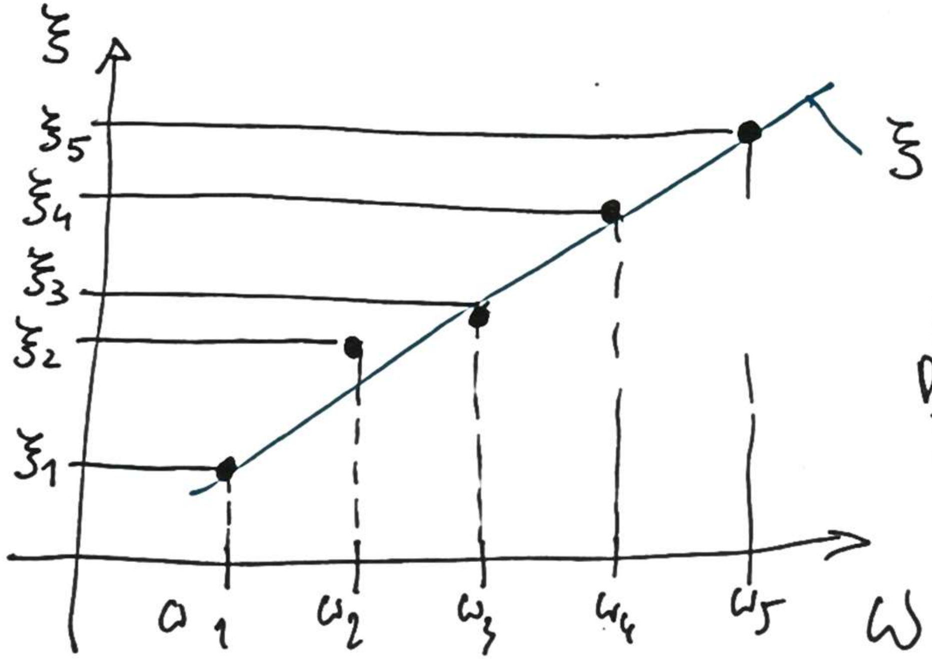
damping ratio:

$$\xi = \frac{\alpha}{2\omega} + \beta \cdot \frac{\omega}{2}$$

$$\xi = \frac{c}{c_{crit}}$$



logarythmic decrement : $\xi_i = \dfrac{\ln \frac{A_i}{A_i'}}{2\pi}$



$$\xi = A \cdot \omega + B$$

↑ FREQUENCY DEPENDENT DAMPING (A)

↑ FREQUENCY INDEPENDENT DAMPING (B)

$$\begin{cases} \dfrac{1}{2\omega_1} \cdot \alpha + \dfrac{\omega_1}{2} \cdot \beta = A \cdot \omega_1 + B \\ \dfrac{1}{2\omega_5} \cdot \alpha + \dfrac{\omega_5}{2} \beta = A \cdot \omega_5 + B \end{cases}$$

$$\alpha = \frac{2\omega_1 \omega_5 B}{\omega_1 + \omega_5}$$

$$\beta = 2\left(A + \frac{B}{\omega_1 + \omega_5}\right)$$

$$[C] = \alpha \cdot [M] + \beta \cdot [K]$$

					LINEAR REGRESSION		A=	2.451E-06	α= 24.75304351 [s]
							B=	1.024E-02	β= 5.52849E-06 [1/s]

TEST no.	f [Hz]	ω [1/s]	A1 [mm]	A1' [mm]	ξ	Rayleigh	mass_part	structural_part
1	200	1256.637	1	0.9	0.016769	0.013323	0.009848923	0.00347365
2	800	5026.548	1	0.87	0.022164	0.016357	0.002462231	0.013894601
3	1600	10053.1	1	0.82	0.031584	0.02902	0.001231115	0.027789201
4	3150	19792.03	1	0.7	0.056767	0.055335	0.000625328	0.05470999
5	5000	31415.93	1	0.57	0.089464	0.087235	0.000393957	0.086841254